Real Options: A Commercial Bank Lending Application

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Abstract

We extend existing real-option theories by incorporating the stochastic interaction between unit price and cost, applied in commercial bank lending. We further empirically examine an implication derived from the model as to the relationship between lending practices in the banking industry and future uncertainties. We focus on lending institutions to analyze the effect of uncertainties on lending (investment) decisions for several reasons. First, it is easy to identify the main sources of uncertainties for the assets and liabilities of the financial institutions - default risk and interest rate changes. Second, the commercial lending institution provides a unique environment in which the correlation between investment costs (liabilities) and output (loans) price is quite high and positive since both depend heavily on interest rates. Finally, bank loans may be subject to a high degree of irreversibility (e.g., substantial loss in defaults).

The real option model explains the relationship between levels of lending, loan-to-assets, and the uncertainties regarding interest income and expenses. The correlation between interest income and loan expenses, in particular, explains cross-sectional loan activities, which confirms the importance of risk management. These results also show that as banks increase one type of risk, e.g., interest rate risk, they decrease another type of risk, e.g., lending risk as measured by loans/assets.

I. Introduction

The banking theory literature has evolved over the years and covered many topics, such as transaction costs (Benston and Smith, 1976), bank production functions (Sealey and Lindley, 1977), informational asymmetries and signaling through self-financing (Leland and Pyle, 1977), and delegated monitoring (Diamond, 1984). We contribute to this literature by examining bank lending from a real-options approach.

The real-options approach to investment under uncertainty has been very useful in explaining investment decisions in many fields. See McDonald and Siegel (1986), Brennan and Schwartz (1985), Majd and Pindyck (1987), Dixit (1989), Pindyck (1991; 1993), Ingersoll and Ross (1992), Grenadier (1995, 1996a, 1996b) among others.[1] The main implication of the approach is that investment decisions can be optimally delayed when there is uncertainty due to an option value to wait. In this paper, we extend existing real-options theories by incorporating the stochastic interaction between unit price and cost, applied in commercial bank lending. We further empirically examine the relationship between lending practices in the banking industry and future uncertainties from the real-options perspective.

We focus on lending institutions to analyze the effect of uncertainties on lending (investment) decisions for several reasons. First, it is easy to identify the main sources of uncertainties for the assets and liabilities of the financial institutions - default risk and interest rate changes. Second, the commercial lending institution provides a unique environment in which the correlation between investment costs (input price) and output price is quite high and positive since both depend heavily on interest rates. The importance of this point will be clearer later. Finally, commercial bank loans may be subject to a high degree of irreversibility (e.g., substantial loss in default), and it may be worthwhile to wait to make lending (investment) decisions.

The overall risk faced by banks depends on the variabilities of assets and liabilities of a bank as well as the correlation between them. Since both assets and liabilities are sensitive to interest rate changes, we argue that a major source of uncertainty in investment decisions (e.g., loan decisions) is interest rate changes.[2] The real-options theory suggests that the level of investment risky assets (e.g., loan portfolio) will be lower (higher) due to the "value to wait" for loan decisions with greater (less) uncertainty, given the same expected return on loans.

We do not imply that risk exposure is purely exogenous in this scenario. We recognize that loan decisions themselves affect risk exposure since they may change the maturity of assets and liabilities or apply hedging strategies. Therefore, we employ a novel approach used in the investment literature (e.g., Fazzari, Hubbard, and Peterson, 1988; Hoshi, Kashyap, and Scharfstein, 1991) to reduce the "endogeneity" problem in our regression specification. Furthermore, we will discuss the implications for risk management and loan decisions.

Section II briefly describes our model of the relationship between loan activities and uncertainties from the real option perspective. Empirical hypotheses and methodologies are discussed in Section III. We summarize and analyze the data and provide empirical results in Section IV. We conclude in Section V.

II. The Model

We assume that a risk neutral bank considers a continuous stream of loan applications instead of a few limited number of applications.[3] Thus, the bank decides to accept loan applications based on the net present value of the loans, which is the discounted net interest revenue which is the interest income minus the interest expense and loan losses. A bank pays C to fund a loan (investment) which yields a cash flow of P (e.g., interest payment) per unit time. Each bank faces uncertainty arising from the stochastic variables, P and C.[4] Therefore, a bank can delay lending from its perspective by rejecting individual applications until the economic condition gets more favorable to the bank.

We assume that the cash flow, P (e.g., interest payment), of the asset (loan) follows a geometric Brownian motion of the form,

$$dP / P = \mu_P dt + \sigma_P dZ_P , \qquad (1)$$

Similarly, the input cost (liability to fund the loan), C, also follows the same process,

$$dC / C = \mu_C dt + \sigma_C dZ_C , \qquad (2)$$

where μ and σ are the expected drift (i.e., growth rate) and diffusion (volatility) of the process, respectively. dZ is the increment of a standard Wiener process. We are interested in the value of the option to make a loan. The option value will determine the optimal tim-

Figure 1: A comparison between traditional real option applications and our application in bank lending

Parameters/environment assumptions	Traditional (capital investments)*	Lending
Underlying assets	Oil or Minerals (stochastic prices)	Loans (stochastic prices)
Exercise Price	Initial investment (constant or stochastic)	Funds to support (stochastic)
Uncertainty	Variance of oil prices Variance of investment costs Correlation between oil and investment costs	Variance of Loan prices Variance of Funds prices Correlation between Loan and Funds prices
Exclusive right To exercise	Varies (depends on sequential investments and market developments)	Varies (depends on loan type and related application fees, current banking relationships and market developments)

* Refer to Pindyck (1991), Sick (1995), Trigeorgis (1996), and Dixit and Pindyck (1994) for an excellent survey of the literature.

ing of investment (loans). We analyze the values of the potential loan and the outstanding loan. By comparing these two values, we can determine the trigger point when a bank makes the actual loans. Let V(P,C) be the value of an opportunity to make a loan. With no actual return on the potential loan, the value of the opportunity to make the loan, V(P,C), is derived from its expected capital gains. Using Ito's lemma and equating the expected capital gain over dt, E[dV(P,C)], to rV(P,C)dt, from the asset equilibrium condition, where r is the bank's constant normal return (or discount rate), we obtain the following differential equation:

$$\frac{1}{2}\sigma_P^2 P^2 V_{PP} + \rho\sigma_P PCV_{PC} + \frac{1}{2}\sigma_C^2 C^2 V_{CC} + \mu_P PV_P + \mu_C CV_C - rV = 0, \quad (3)$$

where ρ is the instantaneous correlation coefficient between the Wiener processes dZ_P and dZ_C that describes price and cost behavior over time.[5] Using the changes of the variables that X(Q)≡V(P,C)/C, Q≡P/C, equation (3) becomes

$$\frac{1}{2}(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2)Q^2 X_{QQ} + (\mu_P + \mu_C)QX_Q - (r - \mu_C)X = 0, \quad (4)$$

Equation (4) has a very important implication for lending decisions under uncertainty. Assuming that interest rates directly affect the stochastic nature of the price of the loan and the funding, the first term in the parenthesis becomes the interest rate risk plus default risk that the bank faces.

The partial differential equation is solved in Appendix 1 and yields:

$$X(Q) = A_0 Q^{-\alpha} + BQ^{\beta}, \quad (5)$$

where

$$-\alpha = \frac{(a-b) - \sqrt{(b-a)^2 - 4ac}}{2a} < 0, \quad (6)$$

$$-\beta = \frac{(a-b) + \sqrt{(b-a)^2 - 4ac}}{2a} > 1, \quad (7)$$

$$a = \frac{1}{2}(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2), b = \mu_P - \mu_C, c = \mu_C - r. \quad (8)$$

and A_0 and B are constants determined from the boundary conditions.

For a very low Q (price-cost ratio), the option to make a loan should be nearly worthless (e.g., deep out-of-money). Thus, we need $A_0 = 0$ in equation (5) to have a solution.

Similarly, let W (P,C) be the value of an "outstanding" loan. In addition to the capital gain, an outstanding loan generates interest income so that the value of an outstanding loan W (P, C) must satisfy the following:

$$\frac{1}{2}(\sigma_P^2 P^2 W_{PP} + \rho\sigma_P\sigma_C PCW_{PC} + \frac{1}{2}\sigma_c^2 C^2 W_{CC} + \mu_p PW_P + \mu_C CW_C - rW + P = 0, \tag{9}$$

Comparing between equation (3) and (9), we realize that the unit income, P, is the only difference. As before, with Y (Q) ≡ W/C and Q ≡ P/C, equation (9) becomes

$$\frac{1}{2}(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_c^2 Q^2 Y_{QQ} + (\mu_p - \mu_C QY_Q - (r - \mu_C)Y + Q = 0 \tag{10}$$

For convergence, we need $r > \mu_P$, and the solution is

$$Y(Q) = AQ^{-\alpha} + B_1 Q^{\beta} + \frac{Q}{r - \mu_P} \tag{11}$$

The last term in the above equation is the expected perpetuity of a unit interest income per unit cost; Q can be interpreted as the effective yield of the loan when the market value of the loan is the same as the cost. Therefore, the remaining terms must be the option value of liquidating the loan. As Q increases, the value of the option decreases so that for a very high Q, the value of the option should be near zero, implying $B_1 = 0$.

To establish the value to wait, define M(Q) = Y(Q) - X(Q). Then, from equations (4) and (10), M(Q) satisfies the following differential equation,

$$\frac{1}{2}(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_c^2)Q^2 M_{QQ} + (\mu_p - \mu_C)QM_Q - (r - \mu_C)M + Q = 0. \tag{12}$$

We evaluate equation (12) at the investment (loan) trigger point, Q_H, to get

$$Q_H = -\frac{1}{2}(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_c^2)Q_H^2 M_{QQ}(Q_H) + (r - \mu_C). \tag{13}$$

since $M(Q_H) = 1$ and $M_Q(Q_H) = 0$. Further, since $M_Q(Q_H) = M_Q(Q_L) = 0$ and $M(Q_H) > M(Q_L)$, M(Q) must be concave at Q_H and convex at Q_L (see Appendix 2 for details).[6] Therefore, $M_{QQ}(Q_H) < 0$. With these derived conditions on parameters, the above equation shows the relationship between the value to wait and the uncertainty (i.e., variance of net interest margin or correlation between loan prices and funding costs). Q_H is the investment (loan) trigger, which is a function of uncertainty, the first term in (13), and the expected net return, the second term. Our focus is on the first term, the effect of uncertainty on the loan trigger, while controlling for the second term, the expected net return.

III. Empirical Hypothesis and "Endogeneity" Issue

A. Empirical Hypothesis

According to Equation (13), the investment (i.e., loan activities) is a function of two important elements: uncertainty indicated by the first term in the equation and the net benefit from loans represented by the second term. Equation (13) says that the greater uncertainty is, the greater the investment trigger is since $M_{QQ}(Q_H) < 0$. In words, the option value of waiting to make loans is higher when the variance of the net interest margin after loan losses is higher, or when the correlation between cost of liabilities and returns from loans is lower, with other parameters constant. If we assume the assets and liabilities are equally sensitive to interest rate changes, or perfectly positively correlated, then it is possible to have an almost perfectly hedged portfolio of assets and liabilities such that there will be little delay in lending decisions due to uncertainty. Then, the loan decision is made on the basis of the expected net worth change alone (i.e., $r-u_c$), without considering the option to wait. Therefore, the implication of the real option model is:

Hypothesis 1: There should be a negative relationship between loan activities and the uncertainty of net interest margin after loan losses.

We will test this implication of the model. We explore the differential cross-sectional effect of the uncertainty (measured by either variance or correlation) on loan activities among banks. Whatever the source may be, each bank may face a different level of uncertainty, depending on the nature of its assets and liabilities. Without uncertainty, the traditional NPV rule suggests that higher NPV induces more loan (investment) activities. Therefore, we use net interest margin and loan loss provisions as control variables in the regression.

The loan loss provision (LLP, hereafter) is sometimes subtracted to obtain the net benefit from loans. This item is not a direct interest expense but reflects a different aspect of the cost of loans, i.e., an expected cost of defaults. Therefore, we add this variable in our regression in order to control for this effect. It is not clear whether the loan loss provision has a positive or negative impact on loan activities. If the loan loss provision is a proxy for the expected cost of default, then it should be reflected in the interest revenue on the loan as part of the interest rate. If the current expected loss is greater than the original expected loss when the loan was made, then a higher (lower) LLP may be associated with a lower (higher) loan-to-assets ratio. In other words, we would expect a negative relationship between LLP and loan-to-assets. On the other hand, the LLP may reflect another aspect of a bank, such as management's attitudes toward loans. A more conservative bank may set a higher LLP for the loans with the same risk. Thus, given the same loan losses, the higher LLP will add to the allowances for loan losses and give the bank more cushion to absorb loan losses. The greater safety cushion may encourage more lending, resulting in a positive relationship between loans/assets and LLP. Empirically, generally we would expect that as the relative amount of loans increases, the loan losses as reflected in LLP would increase.

B. "Endogeneity" Problem

The investment literature (Fazzari, Hubbard, and Peterson, 1988; Hoshi, Kashyap, and Scharfstein, 1991) has recognized the measurement error problem in analyzing the effect of liquidity (independent variable in the regression model) on investment decision (dependent variable). The "omitted variable" problem occurs because liquidity is assumed as exogenous in the regression specification, while in fact, liquidity may proxy for unobservable determinants of investment - the profitability of investment. When a firm's liquidity is high, it is likely to be doing well and so should have good investment opportunity, thus resulting in more investment. The omitted variable problem results in the bias in the liquidity coefficient estimate.

To resolve this bias problem, they divide the sample into two and run separate regressions and examine the difference in beta estimates from each regression. This way, the bias in estimation is cancelled in the difference, representing the true difference assuming that the biases are the same for the two sets of sample. The trick is to separate the sample based on a priori beliefs about how liquidity should affect their investment in each sample. For example, Fazzari, Hubbard, and Peterson (1988) divided firms based on their corporate dividend policies, arguing that firms retaining more earnings are more likely to be liquidity constrained. As expected, investment is found to be more sensitive to liquidity for liquidity constrained firms. Similarly, Hosi, Kashyap, and Scharfstein (1991) use a criterion as to whether Japanese firms are affiliated with major banks or not. Liquidity is expected to be more important for independent (non-affiliated) firms because it is more difficult to raise funds for independent firms especially in Japan.

IV. Data and Empirical Results

A. Data and Summary Statistics

We collected the data on loan activities (loans to assets), the annual amount of loan loss provisions, interest income, and interest expense for the top 100 largest bank holding companies (as of March 31, 1998) for the period of 1987 to 1997. The data were from the National Information Center of the Federal Financial Institutions Examination Council. Only 77 of the top 100 bank holding companies had the necessary information for the entire sample period. Table 1 shows the summary statistics of our sample. As expected, the correlation between interest revenue and interest expense plus loan loss provision is very high on average with the mean value being 93.7%. The average asset size (AA) over the sample period ranges from 2.34 billion to 174 billion dollars with a mean of about 21 billion dollars. The average loan to asset ratio (AL/AA) ranges from 16.3% to 74.3%. The mean loan to asset ratio is 60.2%. Average net interest income is about 790 million dollars. The ratio of the net interest income to assets is 3.85% on average, with a minimum (maximum) of 1.24% (5.01%).

Table 1
Summary Statistics of the Sample

This table contains the summary statistics of the major variables considered for the period of 1987 - 1997. NII is the net interest income, AA is the average asset size for each year, and AL is average loans with all in millions of dollars. ALAA is the ratio of the average loans to average asset size. NIIAA is the ratio of NII to AA. LPAA is the ratio of the average loan loss provision to average size. All these figures are 10-year averages. STD is the standard deviation of net interest margin after loan loss provision, while CORR is the correlation efficient between interest income and interest expense plus loan loss provision for the sample period. Sample size equals 77.

Variables	Mean	Std Error	Minimum	Maximum
NII	790	1102	93	6,674
AA	21,206	29,401	2,350	174,295
STD (%)	0.537	0.363	0.106	1.634
CORR(%)	93.6	8.27	55.5	99.8
ALAA (%)	60.2	11.0	16.3	74.3
NIIAA (%)	3.85	0.65	1.24	5.01
LPAA (%)	0.42	0.206	0.103	0.97

B. Effect of the Uncertainty on Loan Activities (Hypothesis 1)

Following the discussion of the theoretical model in equation (13), the dependent variable is ALAA, average loans/average assets, which is a proxy for the loan trigger, or the loan decision to make more loans. The independent variable of focus is uncertainty, UNC, which is measured by two different proxies, VAR, the variance of net interest margin after loan loss provision, and CORR, the correlation between interest revenue and interest expense plus loan loss provision. The second term in equation (13), expected net return in controlled with two independent variables, NIIAA, the 10-year average of net interest income divided by average assets, and LPAA, the 10-year average of loan loss provision divided by average assets.

We expect VAR to have a negative coefficient, i.e., as the variance or uncertainty increases, the relative amount of lending, ALAA, will decrease. We expect CORR to have a positive coefficient, i.e., as the correlation increases, uncertainty decreases, and the relative amount of lending, ALAA, increases. As discussed earlier, we expect NIIAA to have positive coefficients, i.e., a higher NIIAA indicates more of positive net present value loans and induces more loans. The sign of the coefficient estimate for LPAA is not certain at this point as discussed before. Thus, we employ the following regression model:

$$ALAA_i = \alpha + \beta_1 * UNC_i + \beta_2 * NIIAA_i + \beta_3 * LPAA_i + e_i, \quad i = 1, \ldots, N, \tag{14}$$

where UNC is either CORR or VAR, and

ALAA = 10-year average of average loan-to-asset ratio;

CORR = correlation between interest revenue and interest expense plus loan loss provisions for the 10-year period;

VAR = the variance of the net interest margin after loan loss provision;

NIIAA = 10-year average of net interest income divided by average assets; and

LPAA = 10-year average of loan loss provision divided by average assets.

Table 2 shows the cross-sectional regression results. The estimated coefficient for VAR is significantly negative (t = -2.74), which supports the hypothesis that loan activities are stronger for banks with lower variance for the net interest margin after loan loss provision. That is, the banks with less uncertainty (or risk) tend to accept more loans because of less value to waiting. We obtain a similar result when we use CORR as a measure of uncertainty. The estimated coefficient for CORR is significantly positive (t = 2.37), which is consistent with the previous result and supports that loan activities are stronger for banks with higher correlation between interest revenue and loan expense. The estimated coefficients for NIIAA is also significant and positive (t = 8.392) as expected. Finally, the coefficient estimate for LPAA is also significantly positive, which is consistent with the argument that the loan loss provision may reflect a bank management's aggressiveness (or leniency) towards loans.

Table 2
Regression results for the relationship between loan activities and uncertainty

The coefficient estimates from the cross-section regression using 10-year average values to estimate the effect of uncertainty (UNC) on loan activities (ALAA). CORR or VAR is used to measure UNC where CORR is the correlation between interest revenue and loan expenses for the 10-year period, and VAR is the variance of the net interest margin after loan loss provision. ALAA is the 10-year average loan-to-asset ratio. LPAA is the average loan loss provision divided by average assets. Sample size is 77.

$$ALAA_i = a + b*UNC_i + c*NIIAA_i + d*LPAA_i + e_i$$

	Constant	UNC	NIIAA	LPAA	Adjusted R^2
with VAR	0.1635 (3.19)*	-460.01 (-2.74)*	10.411 (7.30)*	13.6 (2.70)*	57.7%
with CORR	-0.1244 (-1.069)	0.2584 (2.370)**	11.449 (8.392)*	10.412 (2.235)**	56.7%

Note: t-statistics are in the parentheses.
* and ** indicate the significance at the 1% and 5% levels, respectively.

C. Endogeneity Problem and Restriction on Loans

In order to address the endogeneity issue, we adopt the approach used in the investment literature. Note that the source of bias in our situation is slightly different from the one in the investment literature. We face the biased estimation problem due to the endogeneity problem rather than the omitted variable problem. In our case, the endogeneity problem exists since uncertainty (independent variable) is not likely to be exogenous and may be correlated with the disturbance terms, resulting in the bias in the beta coefficient of uncertainty variable. That is, loan activities (loan-to-asset) may affect the uncertainty variable probably via active risk management, leading to its correlation with disturbance.

We divide our sample into two according to loan-to-asset size. The idea is that we expect a different relationship between loan-to-assets and net interest margin variance for high and low loan-to-assets sample. For low loan-to-asset sample, we expect a more significant relationship than for high loan-to-asset sample. We argue that banks with high loan-to-assets may face implicit or explicit restriction on loan activities, and thus their loan decisions may not be as responsive to uncertainty as the model predicts, implying a relatively insignificant beta coefficient in the regression. High loan-to-assets forces banks to be cautious about loan decisions due to high credit risks and tight capital ratio requirements. Table 3 confirms this prediction. The beta coefficient estimate (-570) for uncertainty in the regression with low loan-to-asset sample is significant with a t-value of -2.53. The difference between the two beta coefficients is also significant (t = - 2.77). It is interesting to observe that all other coefficients are also insignificant for high loan-to-asset banks. This result can be interpreted as the credit risk and capital requirement are so severely restrictive for these banks that their loan activities are not sensitive to the change in investment environments.

Table 3
Regression results for the relationship between loan activities and uncertainty (variance): investigation of endogeneity

The coefficient estimates from the cross-section regression using 10-year average values to estimate the effect of uncertainty (VAR) on loan activities (ALAA). VAR is the variance of the net interest margin after loan loss provision. ALAA is the 10-year average loan-to-asset ratio. LPAA is the average loan loss provision divided by average assets. Sample size is 77. The high (low) loan-to-asset sample is the sample with loan-to-asset ratio greater than 0.65 (less than 0.60).

$$ALAA_i = a + b*VAR_i + c*NIIAA_i + d*LPAA_i + e_i$$

	Constant	VAR	NIIAA	LPAA	Adjusted R^2
All (77)	0.1635 (3.19)*	-460.01 (-2.74)*	10.411 (7.30)*	13.6 (2.70)*	57.7%
High (29)	0.5947 (14.35)*	145.15 (1.15)	1.540 (1.51)	4.48 (1.87)	25.8%
Low (28)	0.155 (2.55)**	-570.96 (-2.53)**	8.871 (4.35)*	17.69 (1.99)**	67.6%

Note: t-statistics are in the parentheses.
* and ** indicate the significance at the 1% and 5% levels, respectively.

D. Risk management and correlation between interest revenue and expenses

Although the overall variance of the net interest margin represents the overall uncertainty faced by a bank, the correlation between interest revenue and expenses may be an interesting parameter we need to closely explore further. We argue that the correlation captures the impact of active risk management such as hedging and duration matching more accurately than the overall variance. Therefore, in order to assess the impact of active risk-management factor in making loan decisions, we examine the relationship between loan activities and uncertainty represented by correlation rather than total variance. We expect the endogeneity problem to be more serious in this situation due to closer interaction between loan activities and correlation.

In Table 4, we observe the expected positive relationship between loan activities and correlation parameter for low loan-to-asset sample. However, we obtain the opposite relationship for the high loan-to-asset sample. That is, for the high loan-to-asset sample, we observe lower loan activities for banks with higher correlation between interest revenue and expenses. A plausible interpretation of this result is that the banks in this sample tend to be under pressure to minimize loan activities as they approach a practical upper bound. Therefore, the impact of uncertainty (correlation) on loan activities is expected to be weak, implying a potentially insignificant beta coefficient if there is no endogeneity problem. The insignificant coefficient estimate for NII would be another piece of evidence of insensitivity of loan activities to net interest margin changes.

The significant negative beta may indicate a strong endogeneity effect for the banks facing the upper bound. In sum, the endogeneity (negative) effect dominates over the real option (positive) effect for the high loan-to-asset sample banks, thus resulting in the significant negative beta coefficient. For the low loan-to-asset sample, loan activities are not restricted and can be responsive to correlation factor. Actually, the beta coefficient is significantly positive in Table 4, implying that real-option effect dominates over the endogeneity effect for the low loan-to-asset sample banks.

Table 4
Regression results for the relationship between loan activities and uncertainty (correlation): Effect of Risk management

The coefficient estimates from the cross-section regression using 10-year average values to estimate the effect of uncertainty (CORR) on loan activities (ALAA). CORR is the correlation between interest revenue and loan expenses for the 10-year period. ALAA is the 10-year average loan-to-asset ratio. LPAA is the average loan loss provision divided by average assets. Sample size is 77. The high (low) loan-to-asset sample is the sample with loan-to-asset ratio greater than 0.65 (less than 0.60).

$$ALAA_i = a + b*CORR_i + c*NIIAA_i + d*LPAA_i + e_i$$

	Constant	CORR	NIIAA	LPAA	Adjusted R^2
All (77)	-0.124 (-1.07)	0.258 (2.37)**	11.45 (8.39)*	10.4 (2.23)**	56.7%
High (29)	0.765 (10.04)*	-0.151 (-2.44)**	0.886 (0.956)	5.10 (2.69)*	36.9%
Low (28)	-0.183 (-1.19)	0.304 (2.03)**	9.97 (4.96)*	14.4 (1.62)	65.0%

Note: t-statistics are in the parentheses.
* and ** indicate the significance at the 1% and 5% levels, respectively.

So far we assumed away a fundamental, important issue as to why duration gap exists anyway. If a bank knows the merit of duration match between assets and liabilities, why does duration gap exist across banks? We can address this issue from our model. We emphasize that loan decisions are a function of the net interest margin as well as its variance. That is, loan decisions can be considered as an optimal trade-off between expected return and its risk involved. At times, a bank may forgo risk (increased risk and mismatch) for better returns from loans. Therefore, an existing duration gap is a cumulative result of optimal investment decisions in the mean/variance efficiency sense.

V. Conclusions and Future Research

Our real-options model for lending decisions indicates that the uncertainty regarding interest revenue and loan expenses influences lending decision in addition to the expected net margin on loans. We show empirical evidence consistent with this prediction that the greater uncertainty is, the lower loan activities are. The correlation between interest revenue and loan expenses, in particular, affects the value of waiting on loan decisions. These results also are consistent with the observation that as banks increase one type of risk, e.g., interest rate risk, they decrease another type of risk, i.e., lending risk as measured by loans/assets.

Our model further suggests that since the degree of the correlation between assets (loans) and liabilities (funds) in banking depends on maturity and duration gaps between loans and the liabilities funding them, the value to wait to make loan decisions should depend on those gaps. Therefore, it may be worthwhile to examine cross-sectional relationship between bank lending behavior and duration gap in the future.

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Footnotes

1. McDonald and Siegel (1986) find that an irreversible project under uncertainty is undertaken only when the NPV is sufficiently high. Brennan and Schwartz (1985) value projects in natural resources where the output (oil) price is stochastic. Majd and Pindyck (1987) examine the effects of time to build, opportunity cost and uncertainty in the investment decision based on contingent claims analysis. Dixit (1989) analyzes the optimal entry and exit decision under demand uncertainty and lump-sum costs of adjustment. Pindyck (1993) examines irreversible projects subject to cost uncertainty as in a nuclear plant, development of a new drug, and many R&D projects. Ingersoll and Ross (1992) analyze the effect of interest-rate uncertainty on investment. Also, refer to Pindyck (1991), Sick (1995), Trigeorgis (1996), and Dixit and Pindyck (1994) for an excellent survey of the literature. Very recently, Grenadier (1995, 1996a, 1996b) applies an option-pricing approach to value lease contracts subject to default-risk and to real estate market behavior.

2. A popular measure of a bank's interest rate risk exposure is the average maturity (or duration) difference between its assets and liabilities (see Saunders, 1997 for comprehensive references). The hedging against interest rate fluctuations is possible because of the high correlation between the interest income (output price) and interest expense (input price) in banking organizations (Flannery, 1981).

3. Therefore, a bank behaves as if it had the exclusive right to exercise the option to make loans under uncertainty with a continuous flow of loan applications. The assumption of the exclusive right to exercise is a major difficulty of applying real option models. In traditional real option applications, the degree of the exclusive right depends on sequential investments and market developments. Similarly, in our application in lending, it depends on loan type and its application fee, e.g., the higher the fee or cost to loan applicant, the more exclusive the right would be. Current banking relationships for consumers and commercial firms and a line of credit for commercial borrowers would also affect exclusivity. The stronger the current banking relationship, the more exclusive the right would be. Lander and Pinches (1998) discuss potential difficulties in implementing real option models in reality. Refer to Figure 1 for a comparison between traditional real option applications and our application in bank lending.

4. This is to extend Dixit (1989)'s entry and exit model by incorporating two important stochastic variables in the bank industry - the interest income from loans and the cost of funding them (e.g., interest expenses and loan loss provisions). Grenadier (1995) examines the value of leasing contracts in a similar stochastic environment.

5. McDonald and Siegel (1986) analyzed the optimal investment rule when both the value of a project and the investment cost follow geometric random walks. However, as argued in Pindyck (1991), if the project can be shut down temporarily or permanently due to negative operating profits, the value of a project will not follow a log-normal process, even if the output price does. Thus, a more realistic model would assume that the project's output price follows a geometric random walk, rather than the project value. More recently, Quigg (1993) and Williams (1991) considered real estate investment under two stochastic variables (e.g., unit development costs and unit cash flows to the developed property in Williams and total development costs and the building price in Quigg).

6. Q_H becomes the entry trigger point based on the traditional NPV rule for investment when the first term in equation (13) is zero, i.e., no uncertainty. Q_L is the exit trigger. That is, it is the point when a bank decides to recall or sell off outstanding loans.

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APPENDIX 1

Change the variables following Grenadier (1995):

$$Q \equiv \frac{P}{Q}, X(Q) \equiv \frac{1}{C} V(P,C), Y(Q) \equiv \frac{1}{C} W(P,C)$$

Then, since V(P,I) = IX(Q),

$$V_P = CX_Q \frac{1}{C} = X_Q$$

$$V_C = X + CX_Q (\frac{-P}{C^2}) = X - X_Q \frac{P}{C}$$

$$V_{PP} = \frac{\partial}{\partial p} V_P = X_{QQ} \frac{1}{C}$$

$$V_{CC} = \frac{\partial}{\partial C} V_C = X_Q (\frac{P}{C^2}) - [X_Q (\frac{P}{C^2}) + X_{QQ} \frac{P}{C} (-\frac{P}{C^2})] = X_{QQ} \frac{P^2}{C^3}$$

$$V_{PC} = \frac{\partial}{\partial C} V_P = X_{QQ} (-\frac{P}{C^2})$$

....

Applying the above substitution, equation (3) becomes

$$\frac{1}{2}\sigma_P^2 P^2 X_{QQ} \frac{1}{C} + \rho \sigma_P \sigma_C PCX_{QQ} [(-\frac{P}{C^2})] + \frac{1}{2}\sigma_C^2 C^2 X_{QQ} [\frac{P^2}{C^3}] + \mu_P PX_Q +$$
$$\mu_C C(X - X_Q \frac{P}{C}) - rCX = 0$$

Dividing both sides with C leads to

$$\frac{1}{2}(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2) Q^2 X_{QQ} + (\mu_P - \mu_C) QX_Q - (r - \mu_C) X - 0.$$

Let

$$a = \frac{1}{2}(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2), b = \mu_P - \mu_C, c = \mu_C - r.$$

The solution to the above differential equation is

$$X(Q) = A_0 Q^{-\alpha} + BQ^{\beta}$$

where

$$-\alpha = \frac{(a-b)-\sqrt{(b-a)^2-4ac}}{2a} < 0$$

$$\beta = \frac{(a-b)-\sqrt{(b-a)^2-4ac}}{2a} > 1$$

and A_0 and B are constants to be determined from the boundary conditions.

APPENDIX 2:

To solve the partial differential equations in the main text, we need to impose the two conditions: value-matching and smooth-pasting conditions as follows.

$$(a) V(P_H, C_H) = W(P_H, C_H) - C_H$$

$$(b) V_P(P_H, C_H) = W(P_H, C_H)$$

$$(c) V_C(P_H, C_H) = W_C(P_H, C_H) - 1$$

The condition (a) states that at the investment triggers, P_H and C_H, the value of the loan opportunity equals the value of an outstanding loan after loan creation.

By dividing both sides of the equation by C_H and rearranging the terms, we obtain the following.

$$(a') X(Q_H) = Y(Q_h) - 1$$

$$(b') X_Q(Q_H) - Y_Q(Q_h)$$

$$(c') X(Q_H) - X_Q(Q_h) Q_H = Y(Q_H) - Y_Q(Q_H) Q_H - 1$$